Operational Equivalence and Ontological Underdetermination in Relativistic Gravitation

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Abstract

We show that the full set of classical weak-field tests of General Relativity admits an exact reformulation in terms of two scalar propagation-response fields governing temporal and spatial signal speeds. In this representation, spacetime curvature is not taken as fundamental but emerges as an effective description of an underlying constitutive medium. We demonstrate that, in the screened infrared regime, the difference ("slip") mode becomes parametrically heavy and decouples, yielding a single massless degree of freedom that reproduces the Einstein–Hilbert sector with standard post-Newtonian parameters. Consequently, the empirical successes of General Relativity constrain the observable structure of gravitational phenomena but do not uniquely determine their ontology. Geometry and constitutive-medium descriptions are shown to be operationally equivalent at the tested scales. We discuss the implications for theoretical parsimony, the interpretation of dark sectors, and the general problem of ontological underdetermination in gravitation.

Keywords: general relativity, ontological underdetermination, constitutive fields, effective field theory, post-Newtonian formalism

1 Introduction

General Relativity is widely interpreted as establishing spacetime geometry as the fundamental mediator of gravitation. Empirically, however, observations constrain relations among operationally defined quantities—clock rates, lengths, light propagation, and trajectories—rather than the ontological status of geometry itself. The inference from empirical adequacy to geometric fundamentality is therefore interpretative rather than logically necessary.

This situation exemplifies a broader epistemological pattern. The history of physics demonstrates repeatedly that empirically equivalent formalisms may sustain radically divergent ontological interpretations without loss of predictive power. Schrödinger's wave mechanics and Heisenberg's matrix mechanics initially appeared to constitute distinct physical theories with incompatible ontological commitments—continuous wave evolution versus discrete quantum transitions. Von Neumann's Hilbert space formulation revealed them as mathematically equivalent representations, demonstrating that identical empirical predictions can emerge from disparate conceptual foundations [4].

The present work provides an explicit counterexample to the uniqueness of the geometric interpretation of gravitation. We construct a formulation in which the same observable phenomena arise from two propagation-response fields governing temporal and spatial signal speeds. In the appropriate regime, the theory reduces exactly to the Einstein–Hilbert sector, reproducing all classical tests. The result establishes that the empirical confirmation of General Relativity does not uniquely select a geometric ontology.

It is essential to emphasize that General Relativity itself eliminates gravity as a fundamental force. In the Newtonian framework, gravity is an interaction acting instantaneously at distance between masses. In General Relativity, this force disappears entirely: bodies follow geodesics in curved spacetime, and the apparent acceleration is an artifact of coordinate choice rather than the effect of a real force. The constitutive framework presented here concurs with this elimination. The divergence concerns exclusively the ontological interpretation of the mathematical structure—whether geodesics are primitive geometric objects or optimal paths through a graded propagation medium.

Our aim is not to modify gravitational phenomenology but to analyze the degree to which ontology is underdetermined by the data.

2 Operational Content of Classical Tests

All classical tests of gravitation are ultimately comparisons between operationally defined observables. These may be categorized as follows:

(i) **Clock rates**: gravitational redshift measurements comparing oscillation frequencies of identical systems at different gravitational potentials;

(ii) **Light travel times**: Shapiro delay measurements of round-trip signal propagation past massive bodies;

(iii) **Angular deflections**: gravitational lensing observations of electromagnetic radiation paths;

(iv) **Orbital frequencies**: perihelion precession and binary pulsar timing residuals.

These quantities depend only on the effective propagation structure of signals and test particles. They do not directly probe whether this structure is fundamental geometry or the macroscopic response of deeper degrees of freedom.

Consider the Shapiro time delay. The observable is an excess in round-trip travel time Δt for signals passing near a massive body. The Parametrized Post-Newtonian (PPN) framework expresses this as

$$\Delta t = (1+\gamma_{\text{PPN}})\frac{2GM}{c^3}\ln\frac{4r_1 r_2}{b^2} \tag{1}$$

where M is the source mass, r_1 and r_2 are distances to emitter and receiver, b is the impact parameter, and γ_{PPN} is a dimensionless parameter constrained by Cassini tracking to $\gamma_{\text{PPN}} = 1.00000 \pm 0.00002$ [3].

The experiment measures Δt. It does not measure "curvature" directly. The inference to spacetime geometry requires the additional interpretative step of identifying the effective propagation structure with the metric of a pseudo-Riemannian manifold.

Analogously, gravitational redshift experiments compare clock rates at different altitudes. The observed fractional frequency shift

$$\frac{\Delta \nu}{\nu} = \frac{\Delta \Phi}{c^2} \tag{2}$$

depends on the potential difference $\Delta\Phi$. Whether Φ represents geometric structure or medium response is not determined by the measurement itself.

The empirical content of classical tests therefore constrains an effective metric structure but leaves open the ontology underlying that metric. This is the operational basis for the equivalence we establish.

3 Constitutive Reformulation

We introduce two scalar response fields:

$$\chi(x): \quad \text{temporal response (governs clock rates)} \tag{3}$$

$$\psi(x): \quad \text{spatial response (governs ruler lengths)} \tag{4}$$

These fields encode the propagation properties of the medium. In the presence of a matter distribution with stress-energy tensor $T_{\mu\nu}$, the fields satisfy constitutive relations of the form

$$\nabla^2 \chi = \frac{4\pi G}{c^4}\rho c^2, \quad \nabla^2 \psi = \gamma_{\text{eff}} \frac{4\pi G}{c^4}\rho c^2 \tag{5}$$

where ρ is the matter density and γ_{eff} is a constitutive parameter.

The coordinate velocity of light in this medium reads

$$v_{\text{coord}}(x) = c\sqrt{\frac{1+2\psi(x)/c^2}{1+2\chi(x)/c^2}} \tag{6}$$

The effective metric experienced by test particles and light rays takes the form

$$ds^2 = -\left(1+\frac{2\chi}{c^2}\right)c^2dt^2 + \left(1-\frac{2\psi}{c^2}\right)\delta_{ij}dx^i dx^j \tag{7}$$

This is precisely the weak-field expansion of a general metric with PPN parameter $\gamma_{\text{PPN}} = \psi/\chi$.

The mapping between the field description $\{\chi, \psi\}$ and the metric $g_{\mu\nu}$ is bijective in the weak-field sector. Hence the two descriptions are *mathematically isomorphic* at the level of observables.

We define the gravitational slip parameter

$$\eta \equiv \frac{\psi}{\chi} \tag{8}$$

In General Relativity, $\eta = 1$ identically (single metric potential). The constitutive framework permits $\eta \neq 1$ in principle, though empirical constraints enforce $\eta \to 1$ in the tested regime.

Recovery of classical tests proceeds as follows:

Shapiro delay. The optical path length integral

$$c\,\Delta t = \int (n_{\text{eff}} - 1)\,d\ell = \int \frac{\chi+\psi}{c^2}\,d\ell \tag{9}$$

reproduces Eq. (1) with $(1+\gamma_{\text{PPN}}) = (1+\eta)$ when $\chi = -GM/(c^2 r)$.

Light deflection. Fermat's principle in the graded medium yields deflection angle

$$\alpha = (1+\eta)\frac{2GM}{c^2 b} \tag{10}$$

identical to the PPN prediction.

Gravitational redshift. The frequency shift depends on the temporal field alone:

$$\frac{\Delta\nu}{\nu} = \Delta\chi \tag{11}$$

recovering the standard result.

The constitutive reformulation therefore reproduces all weak-field observables without modification.

4 Infrared Decoupling and Recovery of General Relativity

The equivalence established above operates at the level of observables. We now demonstrate that the geometric description emerges dynamically as the infrared effective theory.

Consider the Lagrangian density for the two-field system:

$$\mathscr{L} = -\frac{c^4}{16\pi G}\left[(\partial\chi)^2 + (\partial\psi)^2\right] - \chi T_{00} - \gamma_{\text{eff}}\psi T_{00} - \frac{\lambda_0}{2} f_{\text{reg}}(X)(\chi-\psi)^2 \tag{12}$$

where $(\partial\phi)^2 \equiv g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$, λ_0 is a dimensionless coupling, and $f_{\rm reg}(X) = 1+\sqrt{1+X^2}$ is a regularized interaction function ensuring well-posed field equations.

Introduce mass eigenstates:

$$\chi_+ \equiv \frac{\chi+\psi}{2} \quad \text{(sum mode)}, \qquad \chi_- \equiv \frac{\chi-\psi}{2} \quad \text{(difference mode)} \tag{13}$$

The kinetic sector transforms as

$$(\partial\chi)^2 + (\partial\psi)^2 = 2\left[(\partial\chi_+)^2 + (\partial\chi_-)^2\right] \tag{14}$$

while the interaction term becomes $(\chi-\psi)^2 = 4\chi_-^2$.

In the eigenstate basis, the Lagrangian reads

$$\mathscr{L} = -\frac{c^4}{8\pi G}\left[(\partial\chi_+)^2 + (\partial\chi_-)^2\right] - (1+\gamma_{\rm eff})\chi_+ T_{00} - (1-\gamma_{\rm eff})\chi_- T_{00} - 2\lambda_0 f_{\rm reg}\chi_-^2 \tag{15}$$

The sum mode χ_+ remains massless and mediates long-range gravitation. The difference mode χ_- acquires an effective mass

$$m_{\rm eff}^2 = \frac{16\pi G\lambda_0}{c^4} f_{\rm reg} \tag{16}$$

In the screened regime where $m_{\rm eff}\ell \gg 1$ (with ℓ the characteristic scale of interest), the heavy mode may be integrated out via its saddle-point equation. The tree-level solution

$$\chi_- \approx \frac{J_-}{m_{\rm eff}^2} \tag{17}$$

is suppressed by $m_{\rm eff}^{-2}$, becoming parametrically small.

Substituting back yields an effective theory for χ_+ alone, with corrections of order $\mathscr{O}(m_{\rm eff}^{-2}\ell^{-2})$. In the limit $m_{\rm eff}\ell \to \infty$:

$$S_{\rm eff}[\chi_+] = S_{\rm EH}[g^{\mu\nu}_{\rm eff}(\chi_+)] + \mathscr{O}(m_{\rm eff}^{-2}) \tag{18}$$

where $g^{\mu\nu}_{\rm eff}$ is the effective metric constructed from the single remaining field with $\chi = \psi = \chi_+$.

General Relativity is therefore not postulated but dynamically generated as the infrared effective theory of the underlying propagation medium.

The slip parameter approaches unity:

$$\eta = \frac{\psi}{\chi} = \frac{\chi_+ - \chi_-}{\chi_+ + \chi_-} \to 1 \quad \text{as} \quad \chi_-/\chi_+ \to 0 \tag{19}$$

This explains why Solar System tests find $\eta \approx 1$ to extraordinary precision: the difference mode has decoupled on those scales.

5 Ontological Underdetermination

We now state the central logical consequence.

If two theoretical frameworks:

(i) produce identical observables in the tested regime,

(ii) share identical post-Newtonian limits,

(iii) differ only in their fundamental variables, then the empirical data cannot uniquely determine which ontology is realized.

General Relativity and the constitutive reformulation satisfy all three conditions. The geometric interpretation posits spacetime as a pseudo-Riemannian manifold whose metric is determined by matter through Einstein's field equations. The constitutive interpretation posits a propagation medium whose response fields $\{\chi, \psi\}$ are determined by stress-energy through constitutive relations. Both predict identical clock rates, light paths, and particle trajectories.

General Relativity therefore constrains structure but not ontology.

This situation is formally analogous to well-established cases in physics:

Domain	**Equivalent Descriptions**
Quantum mechanics	Wave mechanics $\equiv$ Matrix mechanics
Electromagnetism	Field equations $\equiv$ Dielectric medium
Optics	Geometric rays $\equiv$ Refractive index
Thermodynamics	Caloric fluid $\equiv$ Molecular kinetics
Gravitation	Geometric curvature $\equiv$ Constitutive response

The present framework provides the analogous result for relativistic gravitation: empirical success does not entail ontological uniqueness.

6 Remarks on Parsimony

A constitutive interpretation may reduce the number of independent ontological commitments. The standard ΛCDM cosmological model requires:

- Dark matter: an entity of unknown nature comprising $\approx 27\%$ of cosmic energy density;
- Dark energy: an entity of unknown nature comprising $\approx 68\%$ of cosmic energy density;
- Inflation: an additional mechanism with associated inflaton field.

The constitutive framework, in its extended form, requires only the two response fields $\{\chi, \psi\}$ with scale-dependent coupling. Whether this constitutes a parsimony advantage depends on further empirical discrimination.

It is important to note that the constitutive framework does not "modify the laws of gravity." General Relativity itself eliminates gravity as a force—bodies follow geodesics, not trajectories under external influence. The constitutive reformulation concurs with this elimination. The question is not whether to modify gravitational laws (there are none, in the force sense) but which interpretation of the force-free structure is ontologically preferred.

Our claim in this work is limited to logical compatibility, not phenomenological superiority. The two descriptions may diverge outside the screened regime, where the slip mode becomes dynamically relevant. Such extensions, and their observational consequences, lie beyond the present scope.

7 Conclusion

We have established a minimal result: the empirical confirmation of General Relativity does not entail the fundamentality of spacetime geometry. An explicitly equivalent constitutive-medium formulation exists and reproduces all classical tests.

The equivalence is not merely formal. We have shown that the geometric sector emerges dynamically through infrared decoupling of the heavy slip mode, providing a mechanism by which single-potential behavior arises from a two-field substrate. General Relativity occupies the role of a low-energy effective theory, valid below the breakdown scale set by the slip mode mass.

Geometry may therefore be interpreted as emergent rather than primitive. The propagation fields $\{\chi, \psi\}$—governing temporal and spatial signal speeds—provide an alternative foundation with equivalent empirical content at tested scales.

The result contributes to the broader epistemological question of ontological underdetermination in physical theory. Empirical success constrains the relational structure among observables but does not uniquely select the fundamental entities posited by the theory. This pattern, familiar from quantum mechanics and thermodynamics, extends to relativistic gravitation.

Further observational tests may discriminate between the two descriptions beyond the screened regime, where predictions diverge. Until such discrimination is achieved, the choice between geometric and constitutive ontologies remains interpretatively open.

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