Appendix C: Integrating Out the Slip Mode and Recovery of General Relativity in the Screened Infrared Regime

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Abstract

This appendix provides the explicit demonstration that the fundamental medium action $S[\chi, \psi]$ reduces to the Einstein–Hilbert sector in the screened infrared limit, with the slip mode $\chi_- = (\chi - \psi)/2$ becoming parametrically heavy and decoupling. The derivation follows standard effective field theory methodology: identify the heavy degree of freedom, integrate it out at tree level, and verify recovery of the low-energy theory.

Contents

1 Fundamental Lagrangian in Original Variables

From the main text Eq. (130), the weak-field Lagrangian density in the quasi-static sector reads:

$$\mathcal{L} = -\frac{c^4}{16\pi G}\left[g^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi + g^{\mu\nu}\partial_\mu\psi\,\partial_\nu\psi\right] - \chi\,T_{00} - \gamma_{\rm eff}\,\psi\,T_{00} - \frac{\lambda_0}{2}\,f_{\rm reg}(X)\,(\chi-\psi)^2 \tag{1}$$

where the regularized coupling function [Eq. (156)]:

$$f_{\rm reg}(X) = 1 + \sqrt{1+X^2}, \qquad X \equiv -\frac{g^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi}{a_0^2} \tag{2}$$

ensures C^∞ smoothness and well-posed hyperbolic equations.

2 Transformation to Mass Eigenstates

Introduce the sum and difference modes [Eqs. (139)–(140)]:

$$\chi_+ \equiv \frac{\chi+\psi}{2} \qquad \text{(sum mode)}, \qquad \chi_- \equiv \frac{\chi-\psi}{2} \qquad \text{(difference mode)} \tag{3}$$

so that $\chi = \chi_+ + \chi_-$ and $\psi = \chi_+ - \chi_-$.

2.1 Kinetic Sector Transformation

Direct substitution yields:

$$(\partial\chi)^2 + (\partial\psi)^2 = (\partial\chi_+ + \partial\chi_-)^2 + (\partial\chi_+ - \partial\chi_-)^2 = 2\left[(\partial\chi_+)^2 + (\partial\chi_-)^2\right] \tag{4}$$

Consequently, the kinetic normalization transforms as:

$$-\frac{c^4}{16\pi G}\left[(\partial\chi)^2 + (\partial\psi)^2\right] = -\frac{c^4}{16\pi G}\times 2\left[(\partial\chi_+)^2 + (\partial\chi_-)^2\right] = -\frac{c^4}{8\pi G}\left[(\partial\chi_+)^2 + (\partial\chi_-)^2\right] \tag{5}$$

2.2 Interaction Term

The constitutive mass term becomes:

$$(\chi-\psi)^2 = (2\chi_-)^2 = 4\chi_-^2 \tag{6}$$

2.3 Matter Coupling

The source terms decompose as:

$$\chi\,T_{00} + \gamma_{\rm eff}\,\psi\,T_{00} = (1+\gamma_{\rm eff})\,\chi_+\,T_{00} + (1-\gamma_{\rm eff})\,\chi_-\,T_{00} \tag{7}$$

3 Lagrangian in Eigenstate Basis

Collecting terms, the full Lagrangian in the (χ_+, χ_-) basis reads:

$$\boxed{\mathcal{L} = -\frac{c^4}{8\pi G}\left[(\partial\chi_+)^2 + (\partial\chi_-)^2\right] - (1+\gamma_{\rm eff})\,\chi_+\,T_{00} - (1-\gamma_{\rm eff})\,\chi_-\,T_{00} - 2\lambda_0\,f_{\rm reg}(X)\,\chi_-^2} \tag{8}$$

where $(\partial\phi)^2 \equiv g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$.

Physical interpretation: The sum mode χ_+ remains massless and couples to matter with strength $(1+\gamma_{\rm eff})$, mediating the long-range gravitational interaction. The difference mode χ_- acquires an environment-dependent effective mass from the constitutive interaction term, becoming heavy in the screened regime.

4 Effective Mass of the Slip Mode

To extract the effective mass, we compare the interaction term with the canonical massive scalar form. The kinetic term for χ_- has normalization $A \equiv c^4/(8\pi G)$. Rescaling to canonical normalization $\tilde{\chi}_- = \sqrt{2A}\,\chi_-$, the quadratic potential term becomes:

$$-2\lambda_0\, f_{\text{reg}}\, \chi_-^2 = -\frac{\lambda_0\, f_{\text{reg}}}{A}\, \tilde{\chi}_-^2 \tag{9}$$

Comparing with the canonical form $-\frac{1}{2}m_{\text{eff}}^2\, \tilde{\chi}_-^2$, one identifies:

$$\boxed{m_{\text{eff}}^2(g) = \frac{2\lambda_0\, f_{\text{reg}}(g^2/a_0^2)}{A} = \frac{16\pi G\, \lambda_0}{c^4}\, f_{\text{reg}}\left(\frac{g^2}{a_0^2}\right)} \tag{10}$$

where $g = |c^2\nabla\chi|$ denotes the local gravitational acceleration magnitude.

In the screened regime where $g \gg a_0$, the regularization function satisfies $f_{\text{reg}}(g^2/a_0^2) \approx 1 + g^2/a_0^2$, yielding:

$$m_{\text{eff}}^2(g) \simeq \frac{16\pi G\lambda_0}{c^4}\left(1 + \frac{g^2}{a_0^2}\right) \tag{11}$$

The associated screening length is:

$$\ell_{\text{slip}}(g) \equiv m_{\text{eff}}^{-1}(g) = \frac{\ell_0}{\sqrt{f_{\text{reg}}(g^2/a_0^2)}} \tag{12}$$

where the bare screening length [from Eq. (195)]:

$$\ell_0 \equiv \sqrt{\frac{c^4}{16\pi G\lambda_0}} \tag{13}$$

For $\lambda_0 \sim \mathcal{O}(10)$–$\mathcal{O}(100)$ as established in Section 4.3.4 of the main text, one obtains $\ell_0 \sim$ kpc.

5 Tree-Level Integration of the Heavy Mode

In the quasi-static limit, the equation of motion for χ_- obtained from variation of the action is:

$$\frac{c^4}{8\pi G}\, \nabla^2\chi_- - 2\lambda_0\, f_{\text{reg}}\, \chi_- = -\frac{1-\gamma_{\text{eff}}}{2}\, T_{00} \tag{14}$$

Defining the source density:

$$\mathcal{J}_-(x) \equiv \frac{4\pi G(1-\gamma_{\text{eff}})}{c^4}\, T_{00}(x) \tag{15}$$

the field equation takes the standard screened form:

$$\nabla^2\chi_- - m_{\text{eff}}^2\, \chi_- = -\mathcal{J}_- \tag{16}$$

5.1 Saddle-Point Solution

At tree level, for $m_{\text{eff}}\,\ell \gg 1$ where ℓ denotes the characteristic scale of interest, the heavy mode may be integrated out via its saddle-point equation. In this regime the gradient term is subdominant, and the local algebraic solution reads:

$$\chi_-(x) \simeq \frac{\mathcal{J}_-(x)}{m_{\text{eff}}^2(x)} = \frac{4\pi G(1-\gamma_{\text{eff}})}{c^4}\, \frac{T_{00}(x)}{m_{\text{eff}}^2(x)} \tag{17}$$

This contribution is suppressed by $m_{\text{eff}}^{-2} \propto \lambda_{\text{eff}}^{-1}$, becoming parametrically small in the screened regime.

Substituting back into the action generates an effective theory for χ_+ alone, with corrections of order $\mathcal{O}(m_{\text{eff}}^{-2})$.

Loop corrections are suppressed by powers of $(E/m_{\text{eff}})^2$ and therefore remain subleading throughout the screened regime where $m_{\text{eff}} \gg E_{\text{typical}}$.

6 Yukawa Suppression in Real Space

For approximately constant m_{eff} over the region of interest, the exact Green's function solution reads:

$$\chi_-(\mathbf{r}) = \int d^3r' \, \frac{e^{-m_{\text{eff}}|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \, \frac{8\pi G}{c^4} \, \mathcal{J}_-(\mathbf{r}') \tag{18}$$

The difference mode is exponentially suppressed beyond the Yukawa range $\ell_{\text{slip}} = m_{\text{eff}}^{-1}$.

6.1 Solar System Regime

From Eqs. (143)–(150) of the main text, at Earth orbit where $g_\odot/a_0 \sim 1.6 \times 10^8$:

$$\ell_{\text{slip}}^{\text{Solar}} \sim \frac{\ell_0}{1.6 \times 10^8} \sim 6\,\text{m} \tag{19}$$

The slip mode decouples on scales $r \gtrsim 10\,\text{m}$, ensuring $\chi \simeq \psi$ throughout the Solar System to precision $|\chi - \psi|/|\chi| \sim 10^{-21}$ [Eq. (150)].

7 Recovery of $\eta \to 1$ (No-Slip Limit)

The gravitational slip parameter is operationally defined as [Section 4.3.5]:

$$\eta \equiv \frac{\psi}{\chi} = \frac{\chi_+ - \chi_-}{\chi_+ + \chi_-} = 1 - \frac{2\chi_-}{\chi_+} + \mathcal{O}\left(\frac{\chi_-^2}{\chi_+^2}\right) \tag{20}$$

In the screened infrared regime where $\chi_-/\chi_+ \sim \mathcal{O}(m_{\text{eff}}^{-2})$:

$$\boxed{\eta \to 1 \qquad \text{(screened IR)}} \tag{21}$$

The effective metric therefore reduces to the GR weak-field form with a single Newtonian potential $\Phi = -c^2\chi = -c^2\psi$.

8 Regime Criterion and Effective Action

The transition between the two-field unscreened regime and the GR-like screened regime is characterized by:

$$\boxed{r \gg \ell_{\text{slip}}(g) = \sqrt{\frac{c^4}{16\pi G \lambda_0 \, f_{\text{reg}}(g^2/a_0^2)}} \quad \Longrightarrow \quad \chi \simeq \psi, \; \eta \simeq 1} \tag{22}$$

In this limit, the effective action reduces to:

$$S_{\text{eff}}[\chi_+] = S_{\text{EH}}[g_{\mu\nu}^{\text{eff}}(\chi_+)] + \mathcal{O}\left(\frac{1}{m_{\text{eff}}^2 \, \ell^2}\right) \tag{23}$$

where the effective metric is $g_{\mu\nu}^{\text{eff}} = \eta_{\mu\nu} + h_{\mu\nu}(\chi_+)$ with $\chi = \psi = \chi_+$.

9 Interpretation

This appendix makes explicit the effective field theory hierarchy employed throughout the paper. The fundamental medium variables χ, ψ decompose into a massless sum mode χ_+ and a heavy difference mode χ_- whose mass is set by the constitutive response λ_{eff}. Screening corresponds to $m_{\text{eff}}\,\ell \gg 1$ on the probed scales ℓ, whereupon the heavy mode χ_- decouples via standard tree-level integration, and the dynamics collapses to General Relativity (single potential, $\eta \simeq 1$) with parametrically suppressed corrections.

Einstein gravity is therefore not postulated but dynamically generated as the infrared effective theory of the underlying propagation medium.

The geometric description emerges from the collective behavior of the response fields in the same manner that elasticity emerges from atomic interactions or hydrodynamics from molecular dynamics. General Relativity occupies the role of a low-energy effective theory, valid below the breakdown scale $\Lambda_* \sim m_{\text{eff}}$, with the microscopic degrees of freedom (χ, ψ) becoming manifest outside this regime.