

Effective Medium Cosmology: A Minimal Landau–Ginzburg Framework with Kibble–Zurek Freeze-Out

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Abstract

We develop a minimal effective-medium description in which gravitational phenomenology arises from the collective response of two propagation fields. The framework is formulated as a Landau–Ginzburg system with stochastic dynamics consistent with fluctuation–dissipation, and naturally generates a frozen correlation scale through Kibble–Zurek critical slowing down. This leads to a predictive spectrum and correlation structure characterized by two intrinsic lengths only. The model introduces no additional dark-sector degrees of freedom and yields explicit falsifiable consistency relations for growth and lensing observables.

Keywords: effective medium, critical transitions, Kibble–Zurek, consistency relations

1 Introduction

Recent cosmological observations reveal persistent tensions in growth and lensing measurements. KiDS-1000 weak lensing data [4] indicates lower clustering amplitude than predicted by CMB-calibrated Λ CDM. Similar tensions appear in galaxy velocity dispersion versus lensing mass estimates [5]. These systematic discrepancies may indicate missing degrees of freedom in the standard framework.

We develop an alternative effective description based on propagation medium ontology. The universe is modeled as a continuous medium characterized by two independent scalar response fields χ (temporal) and ψ (spatial). Gravitational phenomenology arises from collective dynamics of the order parameter $X = \chi - \psi$ governed by Landau–Ginzburg free energy with stochastic Model A evolution.

Key contributions:

- Minimal rational spectrum fixed by analyticity and two-pole structure
- Frozen correlation scale from Kibble–Zurek critical slowing down

- Four dimensionless consistency relations (CR-1 through CR-4) independent of normalizations
- FDT-constrained amplitude (not freely adjustable)
- No dark-sector degrees of freedom

Roadmap: Section 2 establishes the effective medium framework. Section 3 derives the Kibble–Zurek frozen scale. Section 4 constructs the minimal spectral closure. Section 5 develops observable response fields. Section 6 presents consistency relations and predictions. Section 7 discusses limitations and future tests. Section 8 concludes.

2 Effective Medium Framework

2.1 Order Parameter

The medium is characterized by two independent scalar fields $\chi(\mathbf{x}, t)$ and $\psi(\mathbf{x}, t)$ representing temporal and spatial propagation responses. The fundamental degree of freedom is their misalignment:

$$X(\mathbf{x}, t) \equiv \chi(\mathbf{x}, t) - \psi(\mathbf{x}, t) \quad (1)$$

The sum field $S = \chi + \psi$ is assumed to relax rapidly via thermodynamic closure:

$$S = S_{\text{eq}}(T_{\text{op}}) = S_{\infty} + s_1(T_{\text{op}} - T_{\infty}) \quad (2)$$

where T_{op} is operational temperature. This eliminates gauge freedom.

2.2 Free Energy

The order parameter is governed by Landau–Ginzburg functional:

$$F[X; T_{\text{op}}] = \int d^3x \left[\frac{\kappa}{2} (\nabla X)^2 + \frac{a(T_{\text{op}})}{2} X^2 + \frac{\lambda}{4} X^4 \right] \quad (3)$$

where $\kappa > 0$ is gradient stiffness, $a(T) = a_0(T - T_c)/T_c$ is the control parameter, and $\lambda > 0$ is quartic coupling. The system exhibits second-order transition at T_c .

2.3 Stochastic Dynamics

Non-conserved order parameter evolves via Model A:

$$\frac{\partial X}{\partial t} = -\Gamma \frac{\delta F}{\delta X} + \xi = \Gamma [\kappa \nabla^2 X - a(T)X - \lambda X^3] + \xi \quad (4)$$

where $\Gamma > 0$ is kinetic mobility and ξ is thermal noise satisfying fluctuation–dissipation theorem:

$$\langle \xi(\mathbf{x}, t) \rangle = 0, \quad \langle \xi(\mathbf{x}, t) \xi(\mathbf{x}', t') \rangle = 2\Gamma k_B T_{\text{op}} \delta^{(3)}(\mathbf{x} - \mathbf{x}') \delta(t - t') \quad (5)$$

This constraint fixes the amplitude of fluctuations non-negotiably.

3 Critical Dynamics and Freeze-Out

3.1 Critical Slowing Down

Near the second-order transition, correlation length and relaxation time diverge:

$$\xi(\epsilon) = \xi_0 |\epsilon|^{-\nu}, \quad \tau_{\text{rel}}(\epsilon) = \tau_0 |\epsilon|^{-z\nu} \quad (6)$$

where $\epsilon = (T - T_c)/T_c$ is reduced temperature, and (ν, z) are critical exponents. For Model A: $z \simeq 2$, $\nu \simeq 0.5\text{--}0.63$ (mean-field to 3D Ising).

3.2 Kibble–Zurek Scale

For linear quench $\epsilon(t) \simeq t/\tau_Q$, adiabatic evolution breaks down when:

$$\tau_{\text{rel}}(\epsilon_*) \sim |\epsilon_*| \tau_Q \quad (7)$$

Solving yields freeze-out parameter:

$$|\epsilon_*| \sim \left(\frac{\tau_0}{\tau_Q} \right)^{1/(1+z\nu)} \quad (8)$$

3.3 Emergent Correlation Length

The frozen correlation scale is:

$$\xi_{\text{KZ}} = \xi_0 |\epsilon_*|^{-\nu} = \ell_0 \left(\frac{c_X \tau_Q}{\ell_0} \right)^{\nu/(1+z\nu)} \quad (9)$$

where $\xi_0 = \ell_0$ (unification constraint), $c_X = \sqrt{\kappa\Gamma}$ is medium velocity, and $\tau_0 \sim \ell_0/c_X$. Characteristic wavenumber:

$$k_* = \xi_{\text{KZ}}^{-1} \quad (10)$$

4 Spectral Closure

4.1 Minimal Rational Spectrum

To avoid phenomenological fitting, we adopt the lowest-order rational ansatz uniquely determined by: (i) UV screening above ℓ_0 , (ii) single frozen scale ξ_{KZ} , (iii) analyticity in k^2 , (iv) locality (finite sum of Yukawa kernels).

These constraints select a two-pole form:

$$P_X(k) = A_X \frac{1}{1 + (k\ell_0)^2} \cdot \frac{k^2}{k^2 + k_*^2} \quad (11)$$

This is the lowest-order rational function consistent with the analyticity of the response function and with the presence of exactly two characteristic poles $k_0 = \ell_0^{-1}$ and k_* .

4.2 Amplitude Normalization

From FDT constraint, equilibrium variance:

$$P_X^{\text{eq}}(k) = \frac{k_B T_{\text{op}}}{\kappa k^2 + a} \quad (12)$$

Post freeze-out:

$$A_X \sim \frac{k_B T_{\text{op}}}{\kappa} \ell_0^2 (1 + R^2) \quad (13)$$

where $R = \xi_{\text{KZ}}/\ell_0$. The normalization is determined by thermodynamic parameters within $O(1)$ uncertainties, not freely adjustable.

4.3 Real-Space Correlations

Inverse Fourier transform yields two-Yukawa structure:

$$\xi_X(r) = C_1 \frac{e^{-r/\ell_0}}{r} + C_2 \frac{e^{-r/\xi_{\text{KZ}}}}{r} \quad (14)$$

Amplitudes $C_{1,2}$ determined by (A_X, ℓ_0, k_*) .

4.4 Asymptotic Behavior

The spectrum exhibits:

$$P_X(k) \sim \begin{cases} A_X (k/k_*)^2 & k \ll k_* \\ A_X / (k\ell_0)^2 & k \gg \ell_0^{-1} \end{cases} \quad (15)$$

5 Observable Response

5.1 Screening Filter

Physical observables do not measure X directly but through constitutive screening:

$$X_{\text{eff}}(k) = \frac{X(k)}{1 + (k\ell_0)^2} \quad (16)$$

5.2 Observable Field Y

Linear response relation:

$$Y(k) \equiv \eta(k) - 1 = \alpha \frac{X(k)}{1 + (k\ell_0)^2} \quad (17)$$

where α is constitutive coupling and η is slip parameter.

5.3 Master Evolution Equation

From Eq. (4) and screening:

$$\frac{\partial Y_k}{\partial t} = -\Gamma(\kappa k^2 + a)Y_k + \zeta_k \quad (18)$$

where ζ_k is screened noise.

5.4 Power Spectrum Evolution

Observable spectrum:

$$P_Y(k) = \frac{\alpha^2 A_X}{[1 + (k\ell_0)^2]^3} \cdot \frac{k^2}{k^2 + k_*^2} \quad (19)$$

This is the master formula for all observables linear in X .

6 Consistency Relations and Predictions

6.1 CR-1: Spectral Index Structure

Logarithmic slope $n(k) = d \ln P_Y / d \ln k$ exhibits three regimes:

$$n(k) \approx \begin{cases} +2 & k \ll k_* \\ 0 & k_* \ll k \ll \ell_0^{-1} \\ -6 & k \gg \ell_0^{-1} \end{cases} \quad (20)$$

Test: Verify slope sequence in lensing/clustering data.

6.2 CR-2: Half-Power Point

Define de-screened spectrum $\tilde{P}_Y = P_Y[1 + (k\ell_0)^2]^3$. Then:

$$\frac{\tilde{P}_Y(k_*)}{\tilde{P}_Y(\infty)} = \frac{1}{2} \quad (21)$$

This fixes k_* independent of normalization.

Test: Apply to multiple observables; verify consistency.

6.3 CR-3: Scale Ratio

From Eq. (9):

$$R \equiv \frac{k_0}{k_*} = \left(\frac{\tau_Q}{\tau_0} \right)^{\nu/(1+z\nu)} \quad (22)$$

Test: With ℓ_0 from galactic data and k_* from CMB/BAO, verify τ_Q is cosmologically plausible.

6.4 CR-4: Amplitude Ratio

$$\mathcal{R} \equiv \frac{P_Y(k_0)}{P_Y(k_*)} = f(R) \quad (23)$$

where f is determined by spectral form, independent of A_X .

Test: Measure ratio; compare with prediction from R .

6.5 Prediction: Two-Scale Correlations

Spatial correlation structure:

$$\xi_Y(r) = \sum_{i=0,*} C_i \frac{e^{-r/\xi_i}}{r}, \quad \xi_0 = \ell_0, \quad \xi_* = \xi_{KZ} \quad (24)$$

Kill-switch: If best-fit requires pure power-law or > 2 scales, model excluded.

6.6 Prediction: Localized Non-Gaussianity

From λX^4 term, kurtosis peaks at $k \sim k_*$:

$$K(k) \propto \frac{k^2}{(k^2 + k_*^2)[1 + (k\ell_0)^2]} \quad (25)$$

Kill-switch: If non-Gaussianity uniform or absent at k_* , model excluded.

7 Discussion

7.1 Relation to Analogue Gravity

The framework shares formal structure with BEC analogue systems [6], where collective excitations mimic gravitational degrees of freedom. Unlike geometric approaches, correlations arise from material response rather than spacetime curvature.

7.2 Limitations

Current formulation: (i) linear/quasi-linear regime only, (ii) no explicit baryonic coupling, (iii) toy model critical exponents. Nonlinear structure formation requires numerical simulations beyond present scope.

7.3 Future Observational Tests

Euclid (2026–2030): weak lensing $P_\kappa(k)$, clustering $P_\delta(k)$ to $z \sim 2$

Rubin Observatory (2025–2035): deep galaxy samples, lensing tomography

CMB-S4 (2030s): polarization, lensing reconstruction, non-Gaussianity

Precision: $\Delta k_*/k_* \sim 0.01$ achievable, sufficient for CR tests.

8 Conclusions

The present work establishes internal consistency of the effective-medium formulation. The framework generates predictive structure characterized by dimensionless consistency relations (CR-1 through CR-4) and a minimal set of material parameters $\{\ell_0, \kappa, \Gamma, T_{\text{op}}, \tau_Q, \nu, z, \alpha\}$. No cosmological parameters are introduced.

Confrontation with observational data requires detailed numerical implementation and is left to future studies. The framework provides a testable alternative effective description of cosmological phenomenology.

A Equilibrium Variance Derivation

From linearized dynamics:

$$\frac{dP_k}{dt} = -2\Gamma(\kappa k^2 + a)P_k + 2\Gamma k_B T_{\text{op}}$$

Setting $dP_k/dt = 0$:

$$P_k^{\text{eq}} = \frac{k_B T_{\text{op}}}{\kappa k^2 + a} \quad \blacksquare$$

B Dimensional Estimates

Order-of-magnitude consistency check:

Galactic scale: $\ell_0 \sim 1\text{--}5$ kpc

Cosmological scale: $k_* \sim 0.005\text{--}0.01$ Mpc⁻¹

Ratio: $k_*\ell_0 \sim 10^{-6}\text{--}10^{-4}$

With $\nu/(1+z\nu) \approx 0.28$ (3D Ising):

$$\tau_Q \sim 10^8\text{--}10^{10} \text{ Gyr (cosmologically plausible)}$$

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