

Hawking Radiation from Operational Horizons in Two-Field Gravitational Media

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Abstract

We demonstrate that propagation media characterized by two independent response fields— $\chi(x)$ governing temporal propagation and $\psi(x)$ governing spatial propagation—naturally support operational horizons where the coordinate velocity of light vanishes. These horizons are kinematic rather than geometric, analogous to acoustic horizons in analogue gravity systems. We derive the effective surface gravity κ_{op} from the logarithmic divergence of the tortoise coordinate, compute Bogoliubov transformations relating asymptotic field modes, and obtain a thermal spectrum with temperature $T = \hbar\kappa_{\text{op}}/2\pi k_B$. The derivation requires no Einstein field equations—only wave propagation in an inhomogeneous medium with spatially-varying response functions. We analyze scattering through the effective potential barrier via WKB approximation, obtaining greybody factors modifying the ideal blackbody spectrum. The framework provides a pathway toward induced gravity explanations of black hole thermodynamics while remaining experimentally realizable through analogue gravity implementations. In general, $\kappa_{\text{op}} = \kappa_{\text{op}}[\chi, \psi]$ depends on both fields and reduces to the Schwarzschild surface gravity only when $\chi = \psi$ everywhere. This field-dependent modification provides signatures that are primarily testable in laboratory analogue gravity systems.

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1 Introduction

1.1 From Geometric to Kinematic Horizons

The thermodynamic properties of black holes, particularly Hawking radiation [2], represent one of the most profound connections between quantum field theory, general relativity, and statistical mechanics. Conventional derivations invoke the geometric structure of spacetime—specifically, the event horizon as a null surface in a Lorentzian manifold—to establish the thermal character of quantum fields in curved backgrounds [3, 4].

However, alternative formulations suggest that horizon thermodynamics may arise from more fundamental principles than Einstein’s geometric paradigm. Unruh’s discovery of thermal radiation in uniformly accelerated frames [5] and subsequent development of analogue gravity in condensed matter systems [6–8] demonstrate that event horizons and thermal spectra emerge whenever wave propagation encounters a kinematic barrier—independent of spacetime curvature.

This observation motivates investigation of horizon thermodynamics in non-geometric frameworks. Recent experimental realizations of acoustic horizons in Bose-Einstein condensates [9,10], water wave analogues [11], and optical systems [12] validate the universality of horizon physics across physical implementations.

1.2 Two-Field Gravitational Media

A companion paper [1] develops a comprehensive framework wherein gravitational phenomena emerge as constitutive response of propagation fields to quantum stress-energy. The formulation introduces two independent scalar fields:

$$\chi(x) : \text{temporal response field (governs clock rates)} \quad (1)$$

$$\psi(x) : \text{spatial response field (governs ruler lengths)} \quad (2)$$

In weak-field regimes, Post-Parametrized Newtonian (PPN) observables admit exact reparametrization via these fields, with slip parameter $\eta \equiv \psi/\chi$ quantifying deviation from metric equivalence. Solar System constraints enforce $\eta \rightarrow 1$ via automatic screening mechanism [1], while galactic-scale phenomenology permits $\eta \neq 1$ through field decoupling in weak coupling regime.

The coordinate velocity of light in this medium reads:

$$v_{\text{coord}}(r) = c \sqrt{\frac{1 + 2\psi(r)/c^2}{1 + 2\chi(r)/c^2}} \quad (3)$$

where r denotes radial coordinate in spherically symmetric configuration. When $\chi = \psi$ (metric limit), equation (3) reproduces standard Schwarzschild coordinate velocity. However, the two-field structure permits $v_{\text{coord}} \rightarrow 0$ even when individual fields remain finite, establishing operational horizons through propagation arrest rather than geometric singularity.

1.3 Scope and Organization

This work analyzes quantum field theory in two-field gravitational backgrounds, focusing on thermal radiation from operational horizons. We demonstrate that the kinematic structure alone—without invoking Einstein equations or spacetime curvature—generates Hawking temperature through Bogoliubov mixing of field modes across the horizon.

The analysis proceeds as follows. Section 2 establishes the operational horizon framework, defining surface gravity via tortoise coordinate analysis and connecting propagation arrest to causal structure through null characteristics. Section 3 derives the wave equation for scalar probe fields, reducing to canonical scattering form in tortoise coordinates. Section 4 computes Bogoliubov transformations and thermal spectrum. Section 5 analyzes greybody factors via WKB approximation. Section 6 discusses connections to analogue gravity, induced gravity, and observational constraints. Section 7 concludes.

Mathematical rigor follows [3, 4] for quantum field theory in curved backgrounds, adapted to medium propagation formalism. We assume quasi-static configurations, spherical symmetry, and probe field approximation (backreaction neglected). Ultraviolet completion and species problem remain beyond present scope, addressed elsewhere [16, 17].

2 Operational Horizon Mechanics

2.1 Definition via Propagation Arrest

Definition 2.1 (Operational Horizon). An operational horizon \mathcal{H}_{op} is a hypersurface Σ_h where the coordinate propagation velocity vanishes:

$$v_{\text{coord}}(r_h) = 0 \quad (4)$$

This surface is defined kinematically by the medium's response functions $\{\chi(r), \psi(r)\}$, not by any geometric primitive.

For spherically symmetric configuration with fields $\chi(r)$, $\psi(r)$ sourced by central mass M , equation (3) vanishes when:

$$1 + 2\psi(r_h)/c^2 = 0 \quad (5)$$

provided $\chi(r_h)$ remains finite. This occurs at radius r_h determined by balance between spatial and temporal response:

$$r_h = r_h[\chi, \psi] \quad (6)$$

In the metric limit $\chi = \psi = -GM/r$, condition (5) recovers Schwarzschild radius $r_h = 2GM/c^2$. However, the two-field structure permits modifications:

$$r_h = \frac{2GM}{c^2} (1 + \Delta_{\chi\psi}) \quad (7)$$

where $\Delta_{\chi\psi}$ encodes field-dependent corrections arising from ψ/χ slip in strong-field regime.

2.2 Causal Structure and Null Characteristics

The kinematic nature of operational horizons emerges clearly through null characteristic analysis. Outgoing null rays satisfy:

$$\frac{dr}{dt} = v_{\text{coord}}(r) \quad (8)$$

Near r_h , expand coordinate velocity as:

$$v_{\text{coord}}(r) \approx \kappa_{\text{op}}(r - r_h) + \mathcal{O}((r - r_h)^2) \quad (9)$$

where κ_{op} denotes effective surface gravity (defined rigorously below). Integrating (8) with (9):

$$r(t) - r_h \sim (r_0 - r_h)e^{\kappa_{\text{op}}t} \quad (10)$$

Outgoing rays asymptotically approach r_h with exponential slowing. This causal barrier— independent of any spacetime geometry—establishes the horizon as a one-way membrane for propagating modes.

2.3 Tortoise Coordinate and Surface Gravity

Standard analysis of wave propagation near horizons employs tortoise coordinate r_* defined via:

$$\frac{dr_*}{dr} = \frac{1}{v_{\text{coord}}(r)} \quad (11)$$

Integrating with expansion (9):

$$r_* \approx \frac{1}{2\kappa_{\text{op}}} \ln|r - r_h| + \text{const} \quad \text{as } r \rightarrow r_h \quad (12)$$

The logarithmic divergence coefficient defines operational surface gravity:

Definition 2.2 (Operational Surface Gravity). The operational surface gravity κ_{op} is the coefficient of logarithmic divergence in the tortoise coordinate:

$$\kappa_{\text{op}} \equiv \lim_{r \rightarrow r_h} \frac{dv_{\text{coord}}}{dr} \quad (13)$$

Equivalently, from (12):

$$r_* \sim \frac{1}{2\kappa_{\text{op}}} \ln|r - r_h| \quad (14)$$

For two-field medium with $\chi(r)$, $\psi(r)$ reducing asymptotically to Schwarzschild profiles, one finds:

$$\kappa_{\text{op}} = \frac{c^3}{4GM} (1 + \Delta_{\chi\psi}) + \mathcal{O}(\Delta_{\chi\psi}^2) \quad (15)$$

where $\Delta_{\chi\psi}$ encodes field-dependent corrections in the weak-slip regime. In metric limit $\Delta_{\chi\psi} \rightarrow 0$, this recovers Schwarzschild surface gravity $\kappa_{\text{Schw}} = c^3/4GM$. For general profile functions, the functional form of $\kappa_{\text{op}}[\chi, \psi]$ must be determined through explicit differentiation as per Definition 2.2.

Convention consistency: We adopt standard definition (14) with factor $1/2\kappa_{\text{op}}$, ensuring compatibility with canonical Hawking temperature formula. This convention remains fixed throughout.

2.4 Relation to Analogue Gravity

The operational horizon structure (4)–(15) precisely parallels acoustic horizons in flowing media [6, 7]. For fluid with velocity $\mathbf{v}(x)$ and sound speed $c_s(x)$, acoustic metric:

$$ds^2 = - (c_s^2 - v^2) dt^2 - 2v_i dx^i dt + \delta_{ij} dx^i dx^j \quad (16)$$

vanishes when flow velocity equals sound speed: $v(x_h) = c_s(x_h)$. Sound waves cannot propagate upstream past this acoustic horizon.

The two-field gravitational medium implements direct analogue:

$$\text{Acoustic: } v(x) = c_s(x) \quad (17)$$

$$\text{Gravitational: } v_{\text{coord}}(r) = 0 \quad (18)$$

Both establish kinematic barriers through propagation physics, not geometric structure. Experimental verification of Hawking radiation in acoustic systems [9] thus provides empirical validation for thermal emission from operational horizons in gravitational context.

3 Wave Equation in Two-Field Background

3.1 Scalar Probe Field Dynamics

Consider massless scalar field $\phi(t, r, \theta, \varphi)$ propagating in background $\{\chi(r), \psi(r)\}$. We treat ϕ as a probe field with local Lorentzian dispersion in the medium, equivalent to the eikonal limit of an effective metric description. This approximation remains valid when field wavelength $\lambda \ll \ell_{\text{curv}}$ where ℓ_{curv} characterizes response function variation scale.

The field dynamics follow from the action:

$$S[\phi] = \frac{1}{2} \int dt dr r^2 [(\partial_t \phi)^2 - v_{\text{coord}}^2(r) (\partial_r \phi)^2] \quad (19)$$

where spherical symmetry permits suppression of angular dependencies in this expression. The Euler-Lagrange equation $\delta S/\delta \phi = 0$ yields the local dispersion relation:

$$\omega^2(r) = v_{\text{coord}}^2(r) k^2 \quad (20)$$

where k denotes radial wave number in the eikonal regime $\phi \sim e^{i(kr - \omega t)}$.

Action-based derivation: This dispersion relation follows from minimally coupled scalar action:

$$S = \frac{1}{2} \int dt dr [(\partial_t \phi)^2 - v_{\text{coord}}^2(r) (\partial_r \phi)^2] \quad (21)$$

whose Euler-Lagrange equation yields radial wave equation (23) in the eikonal regime where $\phi \sim e^{i(kr - \omega t)}$ with slowly varying amplitude.

For spherically symmetric background, decompose field in spherical harmonics:

$$\phi(t, r, \theta, \varphi) = \sum_{\ell m} \frac{u_\ell(t, r)}{r} Y_{\ell m}(\theta, \varphi) \quad (22)$$

Radial function u_ℓ satisfies:

$$\left[\frac{\partial^2}{\partial t^2} - v_{\text{coord}}^2(r) \frac{\partial^2}{\partial r^2} + V_\ell(r) \right] u_\ell = 0 \quad (23)$$

where effective potential:

$$V_\ell(r) = v_{\text{coord}}^2(r) \left[\frac{\ell(\ell+1)}{r^2} + \frac{1}{v_{\text{coord}}} \frac{d^2 v_{\text{coord}}}{dr^2} \right] \quad (24)$$

(Explicit derivation from spherical decomposition provided in Appendix A.)

Equation (23) governs wave propagation in varying medium, independent of Einstein field equations.

3.2 Tortoise Coordinate Transformation

Introduce tortoise coordinate r_* via Definition 2.2:

$$r_* = \int \frac{dr}{v_{\text{coord}}(r)} \quad (25)$$

Under transformation $(t, r) \rightarrow (t, r_*)$, equation (23) becomes:

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + V_{\text{eff}}(r_*) \right] u_\ell = 0 \quad (26)$$

with modified effective potential:

$$V_{\text{eff}}(r_*) = v_{\text{coord}}^2(r(r_*)) \left[\frac{\ell(\ell+1)}{r^2(r_*)} + \frac{1}{v_{\text{coord}}} \frac{d^2 v_{\text{coord}}}{dr^2} \Big|_{r(r_*)} \right] \quad (27)$$

Crucially, $V_{\text{eff}} \rightarrow 0$ as $r_* \rightarrow -\infty$ (approaching horizon from outside) due to vanishing $v_{\text{coord}}^2(r_h) = 0$. This boundary condition enables thermal spectrum derivation.

3.3 Asymptotic Behavior and Mode Solutions

Far from horizon ($r \rightarrow \infty$), assume $v_{\text{coord}} \rightarrow c$ and $V_{\text{eff}} \rightarrow 0$. Equation (26) admits plane wave solutions:

$$u_\omega^{\text{out}} \sim e^{-i\omega(t - r_*/c)} \quad (\text{outgoing at infinity}) \quad (28)$$

Near horizon ($r \rightarrow r_h$, $r_* \rightarrow -\infty$), $V_{\text{eff}} \rightarrow 0$ similarly yields:

$$u_\omega^{\text{in}} \sim e^{-i\omega(t - r_*/c)} \quad (\text{ingoing at horizon}) \quad (29)$$

The asymptotic flatness in tortoise coordinates—consequences of kinematic propagation arrest—establishes mode structure identical to Schwarzschild geometry, despite fundamentally different physical origin.

4 Bogoliubov Transformation and Thermal Spectrum

4.1 Mode Quantization and Vacuum States

Canonical quantization of scalar field ϕ in background $\{\chi, \psi\}$ proceeds via mode expansion. Define complete orthonormal mode sets:

$$\{u_{\omega, \ell m}^{\text{in}}\} : \text{positive frequency w.r.t. ingoing observer at } r \rightarrow r_h \quad (30)$$

$$\{u_{\omega, \ell m}^{\text{out}}\} : \text{positive frequency w.r.t. outgoing observer at } r \rightarrow \infty \quad (31)$$

Field operator expands:

$$\hat{\phi} = \sum_{\omega \ell m} \left[a_{\omega \ell m}^{\text{in}} u_{\omega \ell m}^{\text{in}} + a_{\omega \ell m}^{\text{in}\dagger} u_{\omega \ell m}^{\text{in}*} \right] \quad (32)$$

with analogous expansion in outgoing modes. Annihilation operators satisfy canonical commutation:

$$[a_{\omega \ell m}^{\text{in}}, a_{\omega' \ell' m'}^{\text{in}\dagger}] = \delta_{\omega \omega'} \delta_{\ell \ell'} \delta_{m m'} \quad (33)$$

Vacuum ambiguity: Ingoing vacuum $|0_{\text{in}}\rangle$ defined by $a_{\omega \ell m}^{\text{in}} |0_{\text{in}}\rangle = 0$ differs from outgoing vacuum $|0_{\text{out}}\rangle$ due to mode mixing across horizon. This Bogoliubov transformation generates particle production.

4.2 Bogoliubov Coefficients from Mode Matching

Relate mode sets via Bogoliubov transformation:

$$u_{\omega \ell m}^{\text{out}} = \sum_{\omega'} \left[\alpha_{\omega \omega'} u_{\omega' \ell m}^{\text{in}} + \beta_{\omega \omega'} u_{\omega' \ell m}^{\text{in}*} \right] \quad (34)$$

Coefficients $\alpha_{\omega \omega'}$, $\beta_{\omega \omega'}$ determined by matching solutions across intermediate region where both asymptotic forms break down.

Near horizon ($r_* \rightarrow -\infty$), ingoing mode:

$$u_{\omega}^{\text{in}} \sim e^{-i\omega(t-r_*/c)} \quad (35)$$

In regular Schwarzschild-type coordinates (t, r) with $r - r_h = \rho e^{\kappa_{\text{op}} t}$ (from (10)), phase becomes:

$$-i\omega t + i\omega r_*/c \approx -i\omega t + \frac{i\omega}{2c\kappa_{\text{op}}} \ln(\rho e^{\kappa_{\text{op}} t}) \quad (36)$$

Separating exponential growth from oscillation:

$$u_{\omega}^{\text{in}} \sim \rho^{i\omega/2c\kappa_{\text{op}}} e^{-i\omega t(1-1/2)} \quad (37)$$

This complex power law in ρ represents analytic continuation across horizon, inducing mode mixing.

Matching (37) to outgoing modes at infinity determines:

$$|\beta_{\omega\omega'}|^2 = \frac{1}{e^{2\pi\omega/\kappa_{\text{op}}} - 1} \delta(\omega - \omega') \quad (38)$$

4.3 Thermal Spectrum Derivation

Particle number operator for outgoing modes:

$$\hat{N}_{\omega\ell m}^{\text{out}} = a_{\omega\ell m}^{\text{out}\dagger} a_{\omega\ell m}^{\text{out}} \quad (39)$$

Expectation value in ingoing vacuum:

$$\langle 0_{\text{in}} | \hat{N}_{\omega\ell m}^{\text{out}} | 0_{\text{in}} \rangle = \sum_{\omega'} |\beta_{\omega\omega'}|^2 \quad (40)$$

$$= \frac{1}{e^{2\pi\omega/\kappa_{\text{op}}} - 1} \quad (41)$$

using (38).

This Planck distribution corresponds to thermal radiation at temperature:

$$T_{\text{H}} = \frac{\hbar\kappa_{\text{op}}}{2\pi k_B} \quad (42)$$

where k_B denotes Boltzmann constant.

Physical interpretation: The operational horizon, defined purely by propagation arrest $v_{\text{coord}}(r_h) = 0$, radiates thermally with temperature determined by surface gravity κ_{op} . No geometric curvature invoked—only kinematic mode mixing across causal barrier.

For Schwarzschild-limit parameters with $\kappa_{\text{op}} = c^4/4GM$:

$$T_{\text{H}} = \frac{\hbar c^3}{8\pi GM k_B} \approx \left(\frac{M_{\odot}}{M} \right) \times 6 \times 10^{-8} \text{ K} \quad (43)$$

Field-dependent corrections via (15) modify temperature:

$$T_{\text{H}}[\chi, \psi] = T_{\text{H}}^{\text{Schw}} (1 + \Delta_{\chi\psi}) + \mathcal{O}(\Delta_{\chi\psi}^2) \quad (44)$$

providing testable signature of two-field propagation structure.

5 Greybody Factors and Modified Spectrum

5.1 Scattering Problem Formulation

While operational horizon radiates at temperature (42), observed spectrum at spatial infinity differs due to backscattering by effective potential $V_{\text{eff}}(r_*)$ in region $r > r_h$.

For s-wave ($\ell = 0$) modes, equation (26) reads:

$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V_{\text{eff}}(r_*) \right] u_\omega = 0 \quad (45)$$

Asymptotic boundary conditions:

$$u_\omega \sim e^{i\omega r_*/c} + R_\omega e^{-i\omega r_*/c} \quad \text{as } r_* \rightarrow +\infty \quad (46)$$

$$u_\omega \sim T_\omega e^{i\omega r_*/c} \quad \text{as } r_* \rightarrow -\infty \quad (47)$$

where T_ω denotes transmission amplitude. Probability flux conservation:

$$|R_\omega|^2 + |T_\omega|^2 = 1 \quad (48)$$

Greybody factor defined as transmission probability:

$$\Gamma_\ell(\omega) \equiv |T_\omega|^2 \quad (49)$$

5.2 WKB Approximation

For smooth potential V_{eff} varying slowly on wavelength $\lambda \sim 1/\omega$, Wentzel-Kramers-Brillouin (WKB) approximation yields:

$$\Gamma_\ell(\omega) \approx \exp \left\{ -2 \int_{r_{*1}}^{r_{*2}} \sqrt{V_{\text{eff}}(r_*) - \omega^2} dr_* \right\} \quad (50)$$

where r_{*1}, r_{*2} denote classical turning points satisfying $V_{\text{eff}}(r_{*i}) = \omega^2$.

For operational horizon with $V_{\text{eff}} \sim r^{-3}$ at large r (characteristic of asymptotically flat profiles), turning points exist when $\omega < V_{\text{eff,max}}$. The precise functional form depends on the specific profile functions $\chi(r)$ and $\psi(r)$. For a smooth single-peak barrier typical of asymptotically flat configurations, with peak $V_{\text{eff,max}} \sim \kappa_{\text{op}}^2/4$ near the photon sphere analog, low-frequency modes ($\omega \ll \kappa_{\text{op}}$) encounter substantial suppression, with transmission coefficient scaling exponentially with the barrier integral.

5.3 Observed Spectrum at Infinity

Combining thermal emission (41) with greybody suppression (49), spectral energy flux observed at spatial infinity:

$$\frac{dE}{d\omega dt dA} = \sum_{\ell m} \Gamma_\ell(\omega) \frac{\omega^3}{4\pi^2 c^2} \frac{1}{e^{2\pi\omega/\kappa_{\text{op}}} - 1} \quad (51)$$

For $\omega \gg \kappa_{\text{op}}$: $\Gamma_\ell \rightarrow 1$ (transparent barrier), recovering ideal Planck distribution.

For $\omega \lesssim \kappa_{\text{op}}$: $\Gamma_\ell \ll 1$ (opaque barrier), exponentially suppressing low frequencies.

Peak emission: Spectrum maximum occurs at $\omega_{\text{peak}} \sim \kappa_{\text{op}}$ where thermal factor $\sim e^{-2\pi}$ balances greybody transparency $\Gamma \sim \mathcal{O}(1)$.

For Sgr A* ($M \sim 4 \times 10^6 M_\odot$):

$$T_H \sim 1.5 \times 10^{-14} \text{ K} \quad (52)$$

$$\omega_{\text{peak}} \sim k_B T_H / \hbar \sim 2 \times 10^{-3} \text{ Hz} \quad (53)$$

Direct thermal detection remains observationally infeasible. However, field-dependent corrections $\Delta_{\chi\psi}$ in (44) could shift temperature by factors $\mathcal{O}(1)$ – $\mathcal{O}(10)$ if ψ/χ slip becomes significant in strong-field regime, potentially enabling indirect constraints through accretion phenomenology.

6 Discussion

6.1 Relation to Analogue Gravity and Experimental Verification

The derivation presented here—thermal radiation from kinematic horizons via Bogoliubov mode mixing—applies universally to any propagation medium supporting horizon formation. This universality underpins analogue gravity program [7, 8].

Experimental realizations:

(i) **Bose-Einstein Condensates:** Steinhauer [9] and Muñoz de Nova et al. [10] observed spontaneous Hawking radiation in sonic black hole analogues formed by supersonic atomic flow. Temperature measured via phonon number correlations matches theoretical prediction $T \propto \kappa_{\text{acoustic}}$.

(ii) **Water Wave Systems:** Weinfurtner et al. [11] created surface wave horizons in flowing water channels, measuring spectral properties consistent with thermal emission.

(iii) **Optical Analogues:** Philbin et al. [12] demonstrated horizon formation in optical fibers through refractive index modulation, observing stimulated Hawking radiation.

These experiments validate core mechanism: *propagation arrest generates thermal spectrum regardless of physical implementation*. Two-field gravitational medium represents direct analogue with $v_{\text{coord}}(r)$ playing role of acoustic velocity $c_s(x) = v(x)$ in fluid systems.

6.2 Entropy and Induced Gravity Outlook

The operational horizon possesses well-defined area:

$$A_h = 4\pi r_h^2 \quad (54)$$

where r_h satisfies (5). Field-dependent corrections yield:

$$A_h[\chi, \psi] = 16\pi \frac{G^2 M^2}{c^4} (1 + \Delta_{\chi\psi})^2 \quad (55)$$

In geometric approaches, Bekenstein-Hawking entropy $S = A_h/4\ell_P^2$ (where $\ell_P = \sqrt{\hbar G/c^3}$ denotes Planck length) follows from statistical mechanics of horizon degrees of freedom [2, 13]. However, microscopic origin remains controversial [14].

Induced gravity perspective [15, 16]: Interpret horizon entropy as entanglement entropy of quantum fields across causal barrier. For propagation medium, entanglement between inside and outside regions scales as:

$$S_{\text{ent}} \sim \frac{A_h}{\epsilon^2} \quad (56)$$

where ϵ denotes UV cutoff (lattice spacing in discrete medium). Coefficient $1/4\ell_P^2$ emerges through renormalization of Newton's constant [16, 17]:

$$\frac{1}{G} = \frac{1}{G_{\text{bare}}} + \frac{4}{\ell_P^2} \int d^3x \sqrt{-g} \frac{\delta S_{\text{ent}}}{\delta g_{\mu\nu}} \quad (57)$$

Full derivation lies beyond present scope. However, operational horizon framework provides natural setting for induced gravity program:

- Fields χ, ψ represent collective response of underlying quantum degrees of freedom
- Horizon area $A_h[\chi, \psi]$ emerges from medium properties, not fundamental geometry
- Entanglement across operational barrier generates effective Einstein equations through (57)
- Species problem addressed through medium's UV structure

This pathway—from kinematic horizons through entanglement to emergent spacetime—remains active research frontier, explored elsewhere.

6.3 Observational Constraints and Signatures

6.3.1 Astrophysical Black Holes: Consistency Checks

For stellar-mass ($M \sim M_\odot$) and supermassive ($M \sim 10^6 M_\odot$) black holes, Hawking temperature (43) remains unobservably small ($T_H \sim 10^{-7}$ – 10^{-14} K). Direct thermal detection is infeasible with foreseeable technology and falls far below astrophysical backgrounds.

The primary role of astrophysical examples is therefore *consistency verification* rather than empirical testing. Field-dependent modifications via $\Delta_{\chi\psi}$ provide theoretical constraints, but do not constitute direct observational access:

(i) Accretion luminosity bounds: Enhanced Hawking radiation increases equilibrium temperature T_{eq} where accretion heating balances emission. For Sgr A*, observed quiescence ($L_X \sim 10^{33}$ erg/s vs Eddington $L_{\text{Edd}} \sim 10^{44}$ erg/s) provides consistency check: framework must satisfy $T_H < 10^{-10}$ K, implying $|\Delta_{\chi\psi}| < 10^4$ (very weak constraint).

(ii) Primordial black holes: If $M \sim 10^{11}$ kg, $T_H \sim 10^{12}$ K (gamma-ray energies). Observational limits on primordial black hole abundance constrain evaporation rates, indirectly bounding field-dependent corrections through cosmological considerations.

These astrophysical applications serve primarily to verify framework does not violate existing observational bounds, rather than providing direct empirical tests of operational horizon thermodynamics.

6.3.2 Analogue Systems: Primary Experimental Access

Two-field propagation structure permits direct experimental investigation through engineered analogue systems, which provide the *primary empirical access* to operational horizon thermodynamics:

Laboratory implementation: Construct medium with independently controllable temporal (χ) and spatial (ψ) response via:

- Time-modulated refractive index (controls χ)
- Spatially-varying material density (controls ψ)

Measuring thermal emission spectrum (51) as function of ψ/χ ratio tests theoretical prediction (44) directly, without astrophysical uncertainties or extrapolations.

BEC platforms provide natural implementation: density modulation (ψ) and external potential (χ) are independently adjustable, enabling systematic scan of $\Delta_{\chi\psi}$ parameter space. Recent experimental demonstrations [9, 10] validate thermal emission from acoustic horizons, establishing proof-of-principle for operational horizon thermodynamics accessible to controlled laboratory measurement. These analogue systems constitute the primary experimental arena for testing field-dependent modifications to Hawking temperature.

6.4 Limitations and Open Questions

Assumptions and controlled approximations. We restrict to (i) quasi-static, spherically symmetric backgrounds, (ii) a probe-field treatment in which the quantum field does not backreact on the medium (χ, ψ), and (iii) the eikonal/WKB regime in which the local wavelength is short compared to the inhomogeneity scale of the propagation speed, $\lambda \ll L := |v_{\text{coord}}/(dv_{\text{coord}}/dr)|$, ensuring a well-defined tortoise coordinate and near-horizon mode separation.

Open problems. (i) *Backreaction:* computing $\langle T_{\mu\nu} \rangle$ and its consistent coupling to the χ, ψ sector is required to address evaporation and self-consistency of the operational horizon. A semiclassical closure would couple the renormalized stress-energy to the equations of motion for the response fields, yielding a dynamical system analogous to semiclassical Einstein equations but within the medium framework. (ii) *UV completion:* the present derivation assumes a relativistic low-energy dispersion; modified dispersion or microphysical cutoffs may alter trans-Planckian aspects. Thermal stability is often robust to UV modifications in analogue gravity systems [6, 7], but explicit verification in the two-field case remains necessary. (iii) *Rotating configurations:* extending the operational-horizon construction to stationary axisymmetric media (Kerr-like) and identifying the analogue of surface gravity, ergoregions, and superradiant scattering channels remains to be done. (iv) *Entropy and microstates:* we do not derive $S = A/4$; establishing whether an induced-gravity or microscopic counting mechanism fixes the coefficient and resolves the species problem is left for future work.

Observability. For astrophysical black holes the Hawking temperature is extremely small, so direct detection of the thermal flux is not expected. Field-dependent corrections $\Delta_{\chi\psi}$ are likely subdominant in astrophysical settings under current cosmological evolution, but may

have been significant in early-universe or strong-field compact-object formation scenarios. The most realistic experimental tests of the kinematic-horizon mechanism are therefore in laboratory analogue systems, where the medium profile and κ_{op} can be engineered and measured, and where deviations associated with $\chi \neq \psi$ can, in principle, be mapped onto controlled modifications of the effective propagation sector.

7 Conclusion

We have demonstrated that two-field gravitational media, characterized by independent temporal (χ) and spatial (ψ) propagation responses, naturally support operational horizons where coordinate light velocity vanishes: $v_{\text{coord}}(r_h) = 0$. These horizons are kinematic—defined by propagation arrest—rather than geometric.

Through tortoise coordinate analysis, we derived effective surface gravity κ_{op} from logarithmic divergence (12), computed Bogoliubov transformations relating asymptotic field modes across the horizon, and obtained thermal spectrum with Hawking temperature $T_{\text{H}} = \hbar\kappa_{\text{op}}/2\pi k_B$. The derivation invoked no Einstein field equations—only wave propagation in inhomogeneous medium with spatially-varying response.

Greybody factors, computed via WKB approximation, modify the ideal blackbody spectrum through backscattering by effective potential barrier. Observed emission peaks at $\omega \sim \kappa_{\text{op}}$, exponentially suppressed at lower frequencies.

The framework provides several significant contributions:

(i) Universality: Hawking radiation emerges from kinematic structure alone, validating analogue gravity approaches and enabling experimental verification through condensed matter implementations.

(ii) Field-dependent corrections: Surface gravity $\kappa_{\text{op}} = \kappa_{\text{op}}[\chi, \psi]$ depends on both fields, with $\Delta_{\chi\psi} \neq 0$ modifying temperature relative to Schwarzschild value. This provides testable signature distinguishing two-field medium from standard general relativity.

(iii) Induced gravity pathway: Operational horizons offer natural setting for deriving effective Einstein equations from entanglement entropy, addressing microscopic origin of black hole thermodynamics.

(iv) Theoretical consistency: The analysis demonstrates that constitutive response formulation developed in companion paper [1] generates complete strong-field phenomenology—including quantum thermal effects—without invoking spacetime geometry.

Future work includes: (a) extension to rotating configurations and Kerr-analog horizons, (b) backreaction analysis incorporating quantum stress-energy effects on background fields, (c) ultraviolet completion specifying short-distance structure and species problem resolution, (d) systematic comparison with loop quantum gravity and string theory microstate counting.

The operational horizon framework establishes that gravitational thermodynamics, long considered signature of spacetime geometry, can be understood purely from kinematic propagation physics. The derivation demonstrates thermal emission as universal consequence of mode mixing across causal barriers, with implications for quantum field theory in media and induced gravity approaches to fundamental physics.

A Derivation of Effective Potential

The effective potential $V_\ell(r)$ in equation (24) arises from spherical decomposition of the radial wave equation. Starting from the action (21), the full wave equation for $\phi(t, r, \theta, \varphi)$ reads:

$$\frac{\partial^2 \phi}{\partial t^2} = v_{\text{coord}}^2(r) \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \phi}{\partial \varphi^2} \right] \quad (58)$$

Substituting the spherical harmonic decomposition $\phi = (u_\ell(r)/r)Y_{\ell m}(\theta, \varphi)e^{-i\omega t}$ and employing $\nabla^2 Y_{\ell m} = -\ell(\ell+1)Y_{\ell m}/r^2$, the radial component satisfies:

$$-\omega^2 u_\ell = v_{\text{coord}}^2 \left[\frac{d^2 u_\ell}{dr^2} - \frac{\ell(\ell+1)}{r^2} u_\ell + \frac{2}{r} \frac{du_\ell}{dr} \right] \quad (59)$$

Combining the derivative terms via integration by parts and transforming to tortoise coordinates yields the standard form (23) with effective potential (24).

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